



MARKSCHEME

May 2014

MATHEMATICS

Higher Level

Paper 1

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to Scoris instructions and the document “**Mathematics HL: Guidance for e-marking May 2014**”. It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by Scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, eg **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (eg substitution into a formula) and **A1** for using the **correct** values.
- Where the markscheme specifies **(M2)**, **N3**, etc., do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 N marks

*Award N marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in brackets eg (M1), and can only be awarded if correct work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

Follow through (FT) marks are awarded where an incorrect answer from one part of a question is used correctly in subsequent part(s). To award FT marks, there must be working present and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer FT marks.
- If the error leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

If a candidate incorrectly copies information from the question, this is a mis-read (MR). A candidate should be penalized only once for a particular mis-read. Use the MR stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an M mark, but award all others so that the candidate only loses one mark.

- If the question becomes much simpler because of the MR, then use discretion to award fewer marks.
- If the MR leads to an inappropriate value (eg $\sin \theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks (d)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief note written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

$$f'(x) = (2\cos(5x - 3))5 \quad (=10\cos(5x - 3)) \quad \text{AI}$$

Award **AI** for $(2\cos(5x - 3))5$, even if $10\cos(5x - 3)$ is not seen.

10 Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14. Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

SECTION A

1. $P(2) = 24 + 2a + b = 2$, $P(-1) = -3 - a + b = 5$ *M1A1A1*
 $(2a + b = -22, -a + b = 8)$

Note: Award *M1* for substitution of 2 or -1 and equating to remainder, *A1* for each correct equation.

attempt to solve simultaneously
 $a = -10$, $b = -2$

M1
A1

[5 marks]

2. using the sum divided by 4 is 13 *M1*
two of the numbers are 15 *A1*
(as median is 14) we need a 13 *A1*
fourth number is 9 *A1*
numbers are 9, 13, 15, 15

N4
[4 marks]

3. $\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \dots \times \frac{\log 32}{\log 31}$ *M1A1*
 $= \frac{\log 32}{\log 2}$ *A1*
 $= \frac{5 \log 2}{\log 2}$ *(M1)*
 $= 5$ *A1*
hence $a = 5$

[5 marks]

Note: Accept the above if done in a specific base eg $\log_2 x$.

4. $r_1 + r_2 + r_3 = \frac{-48}{5}$ *(M1)(A1)*
 $r_1 r_2 r_3 = \frac{a-2}{5}$ *(M1)(A1)*
 $\frac{-48}{5} + \frac{a-2}{5} = 0$ *M1*
 $a = 50$ *A1*

Note: Award *M1A0M1A0M1A1* if answer of 50 is found using $\frac{48}{5}$ and $\frac{2-a}{5}$.

[6 marks]

5. (a) $\cos x = 2 \cos^2 \frac{1}{2}x - 1$
- $\cos \frac{1}{2}x = \pm \sqrt{\frac{1 + \cos x}{2}}$ *M1*
- positive as $0 \leq x \leq \pi$ *R1*
- $\cos \frac{1}{2}x = \sqrt{\frac{1 + \cos x}{2}}$ *AG*
- [2 marks]*
- (b) $\cos 2\theta = 1 - 2 \sin^2 \theta$ *(M1)*
- $\sin \frac{1}{2}x = \sqrt{\frac{1 - \cos x}{2}}$ *A1*
- [2 marks]*
- (c) $\sqrt{2} \int_0^{\frac{\pi}{2}} \cos \frac{1}{2}x + \sin \frac{1}{2}x \, dx$ *A1*
- $= \sqrt{2} \left[2 \sin \frac{1}{2}x - 2 \cos \frac{1}{2}x \right]_0^{\frac{\pi}{2}}$ *A1*
- $= \sqrt{2} (0) - \sqrt{2} (0 - 2)$ *(A1)*
- $= 2\sqrt{2}$ *A1*
- [4 marks]*
- Total [8 marks]**

6. (a) $x = 1$

AI

[1 mark]

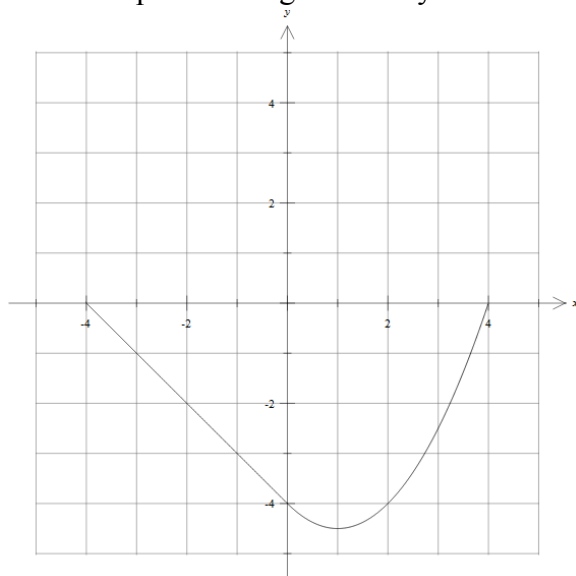
(b) *AI* for point $(-4, 0)$

AI for $(0, -4)$

AI for min at $x = 1$ in approximately the correct place

AI for $(4, 0)$

AI for shape including continuity at $x = 0$



[5 marks]

Total [6 marks]

7. **METHOD 1**

$$AD^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos 60^\circ \quad \text{MI}$$

(or $AD^2 = 1^2 + 3^2 - 2 \times 1 \times 3 \times \cos 60^\circ$)

Note: MI for use of cosine rule with 60° angle.

$$AD^2 = 7 \quad \text{AI}$$

$$\cos \hat{D}AC = \frac{9 + 7 - 4}{2 \times 3 \times \sqrt{7}} \quad \text{MIAI}$$

Note: MI for use of cosine rule involving $\hat{D}AC$.

$$= \frac{2}{\sqrt{7}} \quad \text{AI}$$

METHOD 2

let point E be the foot of the perpendicular from D to AC
 EC = 1 (by similar triangles, or triangle properties) MIAI
 (or AE = 2)

DE = $\sqrt{3}$ and AD = $\sqrt{7}$ (by Pythagoras) (MI)AI

$$\cos \hat{D}AC = \frac{2}{\sqrt{7}} \quad \text{AI}$$

[5 marks]

Note: If first MI not awarded but remainder of the question is correct award M0A0MIAIAI.

8. $\frac{dv}{ds} = 2s^{-3}$ MIAI

Note: Award MI for $2s^{-3}$ and AI for the whole expression.

$$a = v \frac{dv}{ds} \quad \text{(M1)}$$

$$a = -\frac{1}{s^2} \times \frac{2}{s^3} \left(= -\frac{2}{s^5} \right) \quad \text{(A1)}$$

when $s = \frac{1}{2}$, $a = -\frac{2}{(0.5)^5} (= -64) \text{ (ms}^{-2}\text{)}$ MIAI

Note: MI is for the substitution of 0.5 into their equation for acceleration.
 Award MIA0 if $s = 50$ is substituted into the correct equation.

[6 marks]

9. (a) **METHOD 1**

$$\frac{2x}{1+x^4} + \frac{2y}{1+y^4} \frac{dy}{dx} = 0$$

M1A1A1

Note: Award *MI* for implicit differentiation, *AI* for LHS and *AI* for RHS.

$$\frac{dy}{dx} = -\frac{x(1+y^4)}{y(1+x^4)}$$

AI

METHOD 2

$$\begin{aligned} y^2 &= \tan\left(\frac{\pi}{4} - \arctan x^2\right) \\ &= \frac{\tan \frac{\pi}{4} - \tan(\arctan x^2)}{1 + \left(\tan \frac{\pi}{4}\right)\left(\tan(\arctan x^2)\right)} \\ &= \frac{1 - x^2}{1 + x^2} \end{aligned}$$

(M1)

$$2y \frac{dy}{dx} = \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2}$$

MI

$$2y \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = -\frac{2x}{y(1+x^2)^2}$$

AI

$$\left(= \frac{2x\sqrt{1+x^2}}{\sqrt{1-x^2}(1+x^2)^2} \right)$$

[4 marks]

continued ...

Question 9 continued

$$(b) \quad y^2 = \tan\left(\frac{\pi}{4} - \arctan\frac{1}{2}\right) \quad (M1)$$

$$= \frac{\tan\frac{\pi}{4} - \tan\left(\arctan\frac{1}{2}\right)}{1 + \left(\tan\frac{\pi}{4}\right)\left(\tan\left(\arctan\frac{1}{2}\right)\right)} \quad (M1)$$

Note: The two *MI*s may be awarded for working in part (a).

$$= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3} \quad AI$$

$$y = -\frac{1}{\sqrt{3}} \quad AI$$

substitution into $\frac{dy}{dx}$

$$= \frac{4\sqrt{6}}{9} \quad AI$$

Note: Accept $\frac{8\sqrt{3}}{9\sqrt{2}}$ etc.

[5 marks]

Total [9 marks]

10. $\sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{4}{9}$ *(M1)(A1)*

using $\sin^2 x + \cos^2 x = 1$ *(M1)*

$$2 \sin x \cos x = -\frac{5}{9}$$

using $2 \sin x \cos x = \sin 2x$ *(M1)*

$$\sin 2x = -\frac{5}{9}$$

$\cos 4x = 1 - 2 \sin^2 2x$ *M1*

Note: Award this *M1* for decomposition of $\cos 4x$ using double angle formula anywhere in the solution.

$$= 1 - 2 \times \frac{25}{81}$$

$$= \frac{31}{81}$$

A1

[6 marks]

SECTION B

11. (a) $f'(x) = \frac{x \times \frac{1}{x} - \ln x}{x^2}$ *M1A1*
 $= \frac{1 - \ln x}{x^2}$ *AG*
[2 marks]

(b) $\frac{1 - \ln x}{x^2} = 0$ has solution $x = e$ *M1A1*
 $y = \frac{1}{e}$ *A1*
 hence maximum at the point $\left(e, \frac{1}{e} \right)$
[3 marks]

(c) $f''(x) = \frac{x^2 \left(-\frac{1}{x} \right) - 2x(1 - \ln x)}{x^4}$ *M1A1*
 $= \frac{2 \ln x - 3}{x^3}$

Note: The *M1A1* should be awarded if the correct working appears in part (b).

point of inflexion where $f''(x) = 0$ *M1*
 so $x = e^{\frac{3}{2}}$, $y = \frac{3}{2}e^{-\frac{3}{2}}$ *A1A1*
 C has coordinates $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}} \right)$
[5 marks]

(d) $f(1) = 0$ *A1*
 $f'(1) = 1$ *(A1)*
 $y = x + c$ *(M1)*
 through $(1, 0)$
 equation is $y = x - 1$ *A1*
[4 marks]

continued ...

Question 11 continued

(e) **METHOD 1**

$$\text{area} = \int_1^e x - 1 - \frac{\ln x}{x} dx$$

M1A1A1

Note: Award **MI** for integration of difference between line and curve, **AI** for correct limits, **AI** for correct expressions in either order.

$$\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} (+c)$$

(M1)AI

$$\int (x-1) dx = \frac{x^2}{2} - x (+c)$$

AI

$$= \left[\frac{1}{2}x^2 - x - \frac{1}{2}(\ln x)^2 \right]_1^e$$

$$= \left(\frac{1}{2}e^2 - e - \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right)$$

$$= \frac{1}{2}e^2 - e$$

AI

METHOD 2

$$\text{area} = \text{area of triangle} - \int_1^e \frac{\ln x}{x} dx$$

M1A1

Note: **AI** is for correct integral with limits and is dependent on the **MI**.

$$\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} (+c)$$

(M1)AI

$$\text{area of triangle} = \frac{1}{2}(e-1)(e-1)$$

M1A1

$$\frac{1}{2}(e-1)(e-1) - \left(\frac{1}{2} \right) = \frac{1}{2}e^2 - e$$

AI

[7 marks]

Total [21 marks]

12. (a) $|\vec{OA}| = |\vec{CB}| = |\vec{OC}| = |\vec{AB}| = 6$ (therefore a rhombus) *A1A1*

Note: Award *A1* for two correct lengths, *A2* for all four.

Note: Award *A1A0* for $\vec{OA} = \vec{CB} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ or $\vec{OC} = \vec{AB} = \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix}$ if no magnitudes are shown.

$$\vec{OA} \cdot \vec{OC} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix} = 0 \text{ (therefore a square)} \quad \text{AI}$$

Note: Other arguments are possible with a minimum of three conditions.

[3 marks]

(b) $M \left(3, -\frac{\sqrt{24}}{2}, \frac{\sqrt{12}}{2} \right) (= (3, -\sqrt{6}, \sqrt{3}))$ *A1*

[1 mark]

(c) **METHOD 1**

$$\vec{OA} \times \vec{OC} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix} = \begin{pmatrix} 0 \\ -6\sqrt{12} \\ -6\sqrt{24} \end{pmatrix} = \begin{pmatrix} 0 \\ -12\sqrt{3} \\ -12\sqrt{6} \end{pmatrix} \quad \text{M1A1}$$

Note: Candidates may use other pairs of vectors.

equation of plane is $-6\sqrt{12}y - 6\sqrt{24}z = d$
 any valid method showing that $d = 0$ *M1*
 $\Pi : y + \sqrt{2}z = 0$ *AG*

METHOD 2

equation of plane is $ax + by + cz = d$
 substituting O to find $d = 0$ *(M1)*
 substituting two points (A, B, C or M) *M1*
eg
 $6a = 0, -\sqrt{24}b + \sqrt{12}c = 0$ *A1*
 $\Pi : y + \sqrt{2}z = 0$ *AG*

[3 marks]

continued ...

Question 12 continued

(d)
$$\mathbf{r} = \begin{pmatrix} 3 \\ -\sqrt{6} \\ \sqrt{3} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ \sqrt{2} \end{pmatrix}$$
 A1A1A1

Note: Award *A1* for $\mathbf{r} =$, *A1A1* for two correct vectors.

[3 marks]

- (e) Using $y = 0$ to find λ *M1*
 Substitute their λ into their equation from part (d) *M1*
 D has coordinates $(3, 0, 3\sqrt{3})$ *A1*

[3 marks]

- (f) λ for point E is the negative of the λ for point D *(M1)*

Note: Other possible methods may be seen.

E has coordinates $(3, -2\sqrt{6}, -\sqrt{3})$ *A1A1*

Note: Award *A1* for each of the y and z coordinates.

[3 marks]

(g) (i)
$$\vec{DA} \cdot \vec{DO} = \begin{pmatrix} 3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} = 18$$
 M1A1

$$\cos \hat{ODA} = \frac{18}{\sqrt{36}\sqrt{36}} = \frac{1}{2}$$
 M1

hence $\hat{ODA} = 60^\circ$ *A1*

Note: Accept method showing OAD is equilateral.

- (ii) OABCDE is a regular octahedron (accept equivalent description) *A2*

Note: *A2* for saying it is made up of 8 equilateral triangles
 Award *A1* for two pyramids, *A1* for equilateral triangles.
 (can be either stated or shown in a sketch – but there must be clear indication the triangles are equilateral)

[6 marks]

Total [22 marks]

13. (a) $r = 1 + i$ (A1)
 $u_4 = 3(1 + i)^3$ (M1)
 $= -6 + 6i$ (A1)
 [3 marks]

(b) $S_{20} = \frac{3((1+i)^{20} - 1)}{i}$ (M1)
 $= \frac{3((2i)^{10} - 1)}{i}$ (M1)

Note: Only one of the two *M1*s can be implied. Other algebraic methods may be seen.

$= \frac{3(-2^{10} - 1)}{i}$ (A1)
 $= 3i(2^{10} + 1)$ (A1)
 [4 marks]

- (c) (i) **METHOD 1**
 $v_n = (3(1+i)^{n-1})(3(1+i)^{n-1+k})$ (M1)
 $9(1+i)^k (1+i)^{2n-2}$ (A1)
 $= 9(1+i)^k ((1+i)^2)^{n-1} (= 9(1+i)^k (2i)^{n-1})$
 this is the general term of a geometrical sequence (RIAG)

Notes: Do not accept the statement that the product of terms in a geometric sequence is also geometric unless justified further.
 If the final expression for v_n is $9(1+i)^k (1+i)^{2n-2}$ award *M1A1R0*.

METHOD 2

$\frac{v_{n+1}}{v_n} = \frac{u_{n+1}u_{n+k+1}}{u_n u_{n+k}}$ (M1)
 $= (1+i)(1+i)$ (A1)
 this is a constant, hence sequence is geometric (RIAG)

Note: Do not allow methods that do not consider the general term.

- (ii) $9(1+i)^k$ (A1)
 (iii) common ratio is $(1+i)^2 (= 2i)$ (which is independent of k) (A1)
 [5 marks]

continued ...

Question 13 continued

(d) (i) **METHOD 1**

$$w_n = |3(1+i)^{n-1} - 3(1+i)^n| \quad \text{MI}$$

$$= 3|1+i|^{n-1} |1-(1+i)| \quad \text{MI}$$

$$= 3|1+i|^{n-1} \quad \text{AI}$$

$$\left(= 3(\sqrt{2})^{n-1} \right)$$

this is the general term for a geometric sequence **RIAG**

METHOD 2

$$w_n = |u_n - (1+i)u_n| \quad \text{MI}$$

$$= |u_n| |-i|$$

$$= |u_n| \quad \text{AI}$$

$$= |3(1+i)^{n-1}|$$

$$= 3|(1+i)^{n-1}| \quad \text{AI}$$

$$\left(= 3(\sqrt{2})^{n-1} \right)$$

this is the general term for a geometric sequence **RIAG**

Note: Do not allow methods that do not consider the general term.

(ii) distance between successive points representing u_n in the complex plane forms a geometric sequence **RI**

Note: Various possibilities but must mention distance between successive points.

[5 marks]

Total [17 marks]